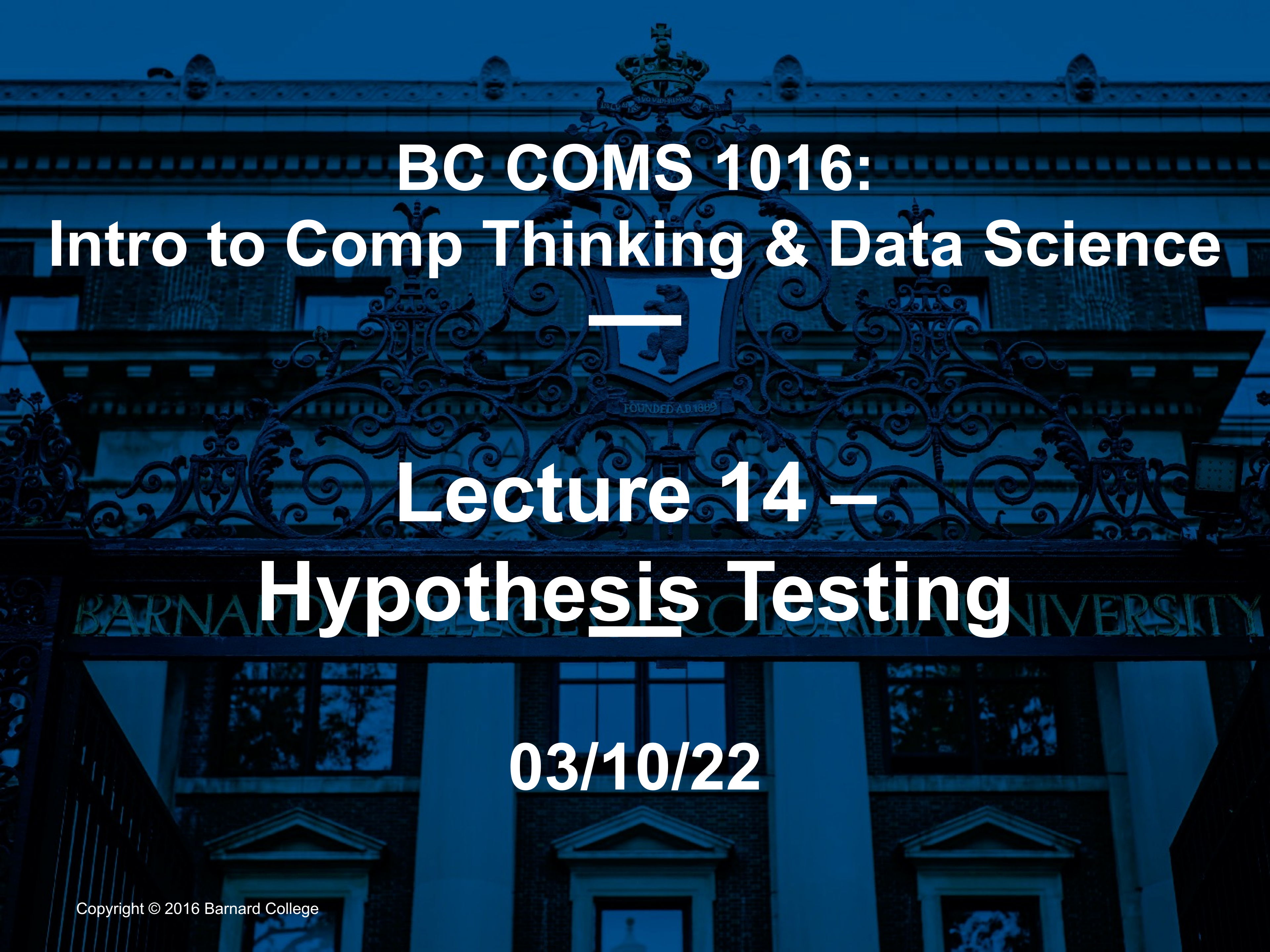

BC COMS 1016: Intro to Comp Thinking & Data Science

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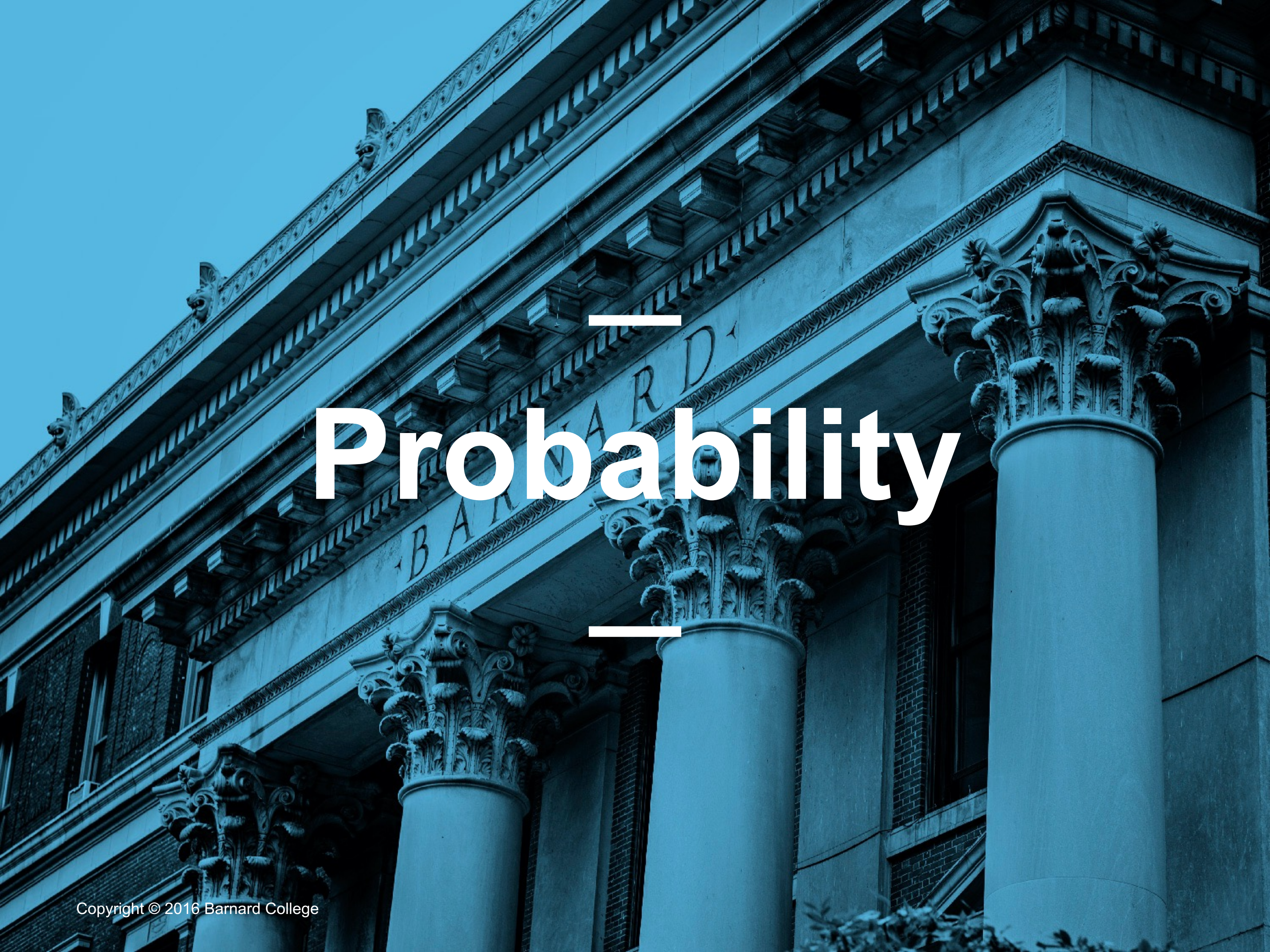
Lecture 14 – Hypothesis Testing

03/10/22

Announcements

- Lab 05 - Assessing Models: Examining the Therapeutic Touch
  - Due tomorrow (03/11)
- HW05 - Probability, Simulation, Estimation, and Assessing Models
  - Due tomorrow (03/11)
- Project 1:
  - Almost done grading
- Project 2 (midterm):
  - Released after spring break

Flexible grading policy for three projects



Probability

Complement: be careful

- A = the event of sampling (with replacement) 5 aces in a row from a deck of card. P(A) = ?
  - $\frac{1}{52} \times \frac{1}{52} \times \frac{1}{52} \times \frac{1}{52} \times \frac{1}{52}. = \frac{1}{52}^5$

- What is the complement of A?
  1. Drawing 5 cards and never getting an ace
  2. Drawing 5 cards and not getting 5 aces

Complement: be careful

- B = the event of sampling (with replacement) 5 cards and never getting an ace. P(B) = ?
  - $\frac{48}{52} \times \frac{48}{52} \times \frac{48}{52} \times \frac{48}{52} \times \frac{48}{52}. = \frac{48^5}{52}$

P(A) = $\frac{1^{5}}{52}$; P(B) = $\frac{48^5}{52}$

- Is P(A) = 1 – P(B)?
  - P(A) = $\frac{1^{5}}{52} \cong \frac{1}{380M}$
  - P(B) = $\frac{48^5}{52} \cong \frac{254M}{380M}$

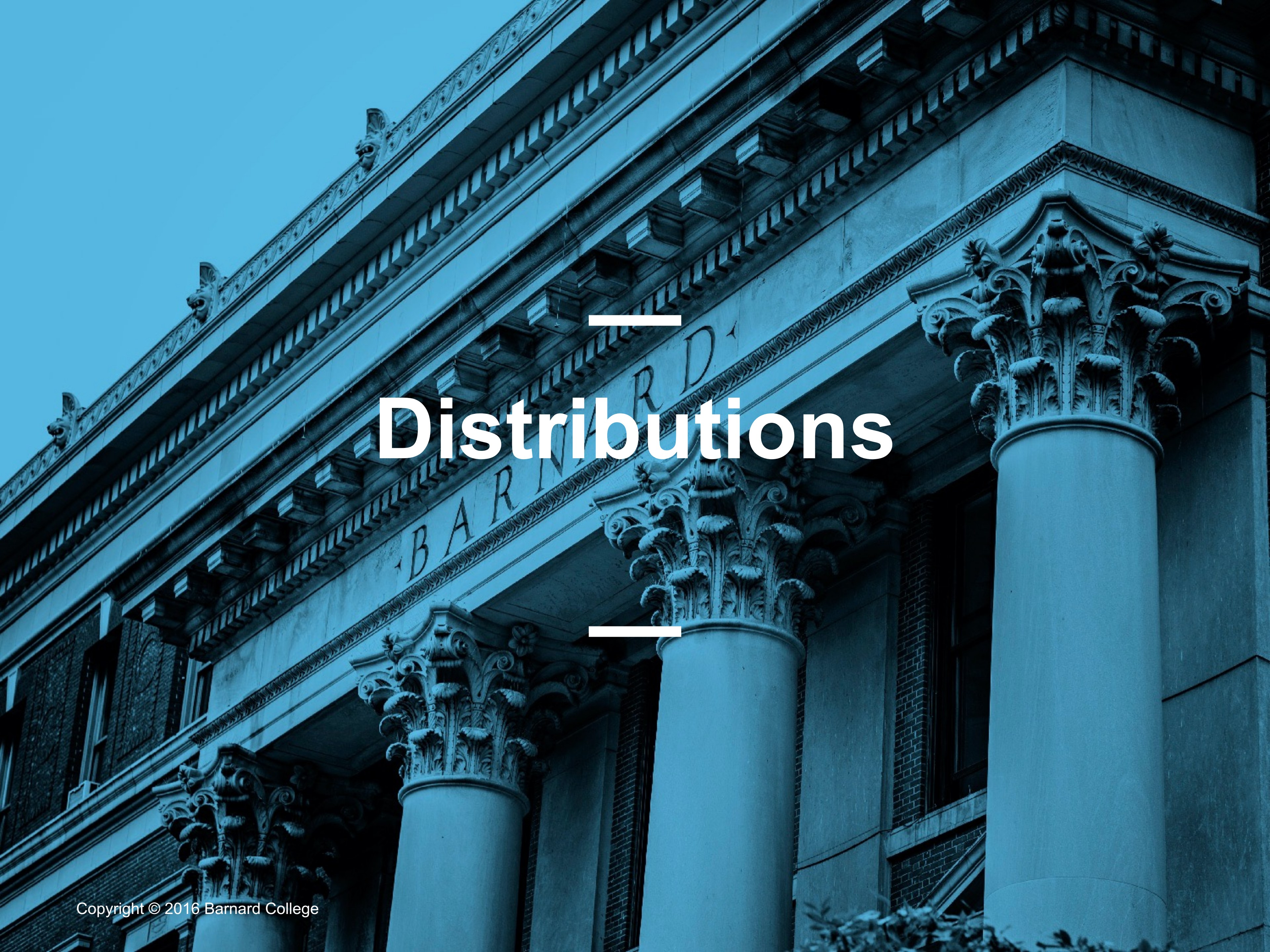
Complement: be careful

- A = the event of sampling (with replacement) 5 aces in a row from a deck of card. P(A) = ?
  - $\frac{1}{52} \times \frac{1}{52} \times \frac{1}{52} \times \frac{1}{52} \times \frac{1}{52}. = \frac{1}{52}^{5}$

- The complement of A is:
  1. Drawing 5 cards and never getting an ace
  2. P(not A) = $1 - \frac{1}{52}^{5} \cong \frac{380M - 1}{380M}$

Probability & Sampling

Distributions



Large Random Samples

A Statistic

Why bother sampling?

Probability

Statistics

Sampling

Inference

- **Statistical Inference:**
  - Making conclusions based on data in random samples

- **Example:**

fixed

  - Use the data to guess the value of an unknown number

Depends on the random sample

  - Create an **estimate** of an unknown quantity

Terminology

- **Parameter**
  - Numerical quantity associated with the population

- **Statistic**
  - A number calculated from the sample

- A statistic can be used as an **estimator** of a parameter

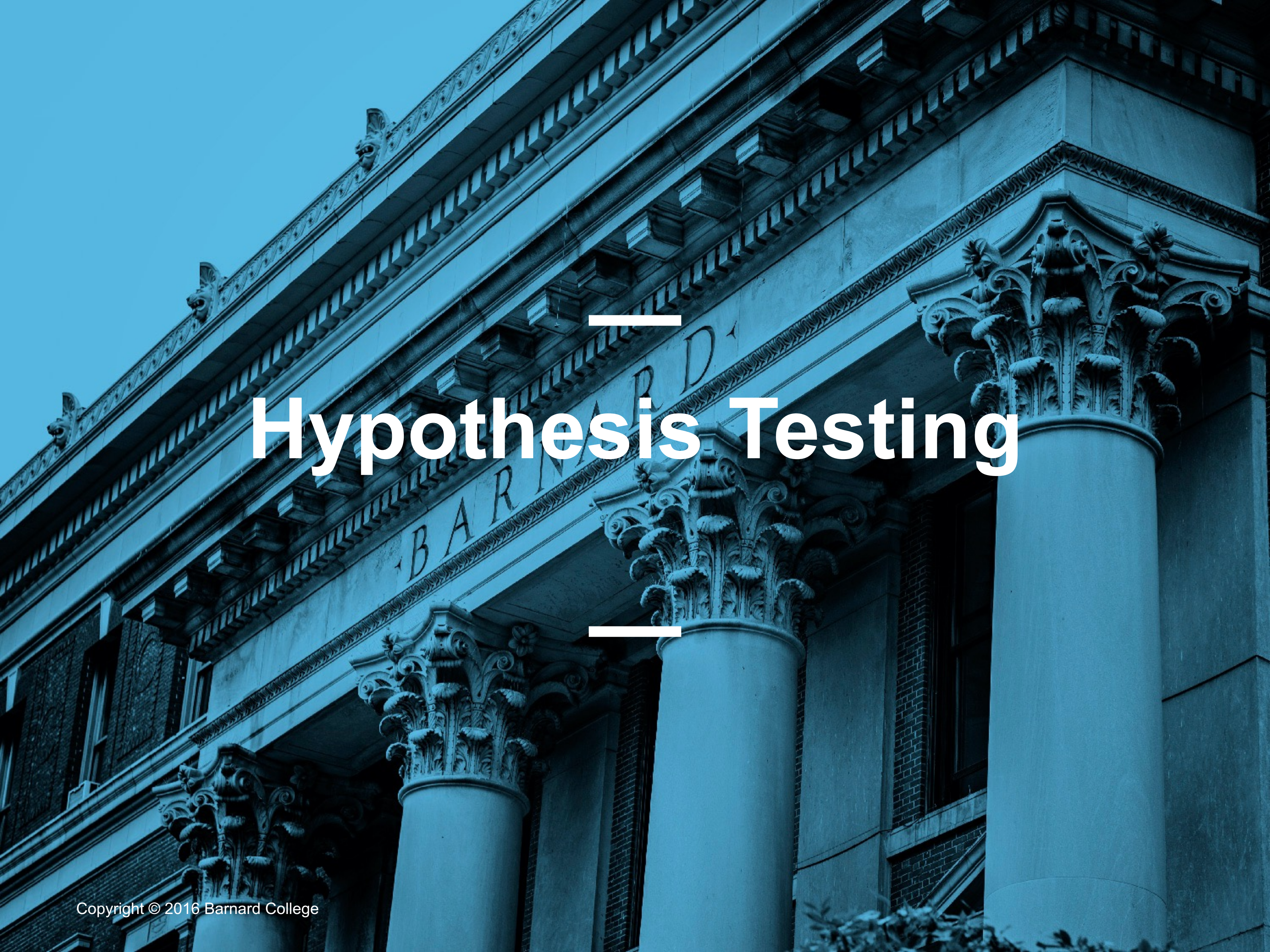
Probability distribution of a statistic

- Values of a statistic vary because random samples vary
- "Sampling distribution" or "probability distribution" of the statistic:
  - All possible values of a statistic
  - and all corresponding probabilities
- Can be hard to calculate:
  - Either have to do math
  - Or generate all possible samples and calculate the statistic based on the each sample

Empirical Distribution of a Statistic

- Based on simulated values of a statistic
- Consists of all observed values of the statistic,
- and the proportion of times each value appeared

- Good approximation to the probability distribution of a statistic
  - If the number of repetitions in the simulation is large

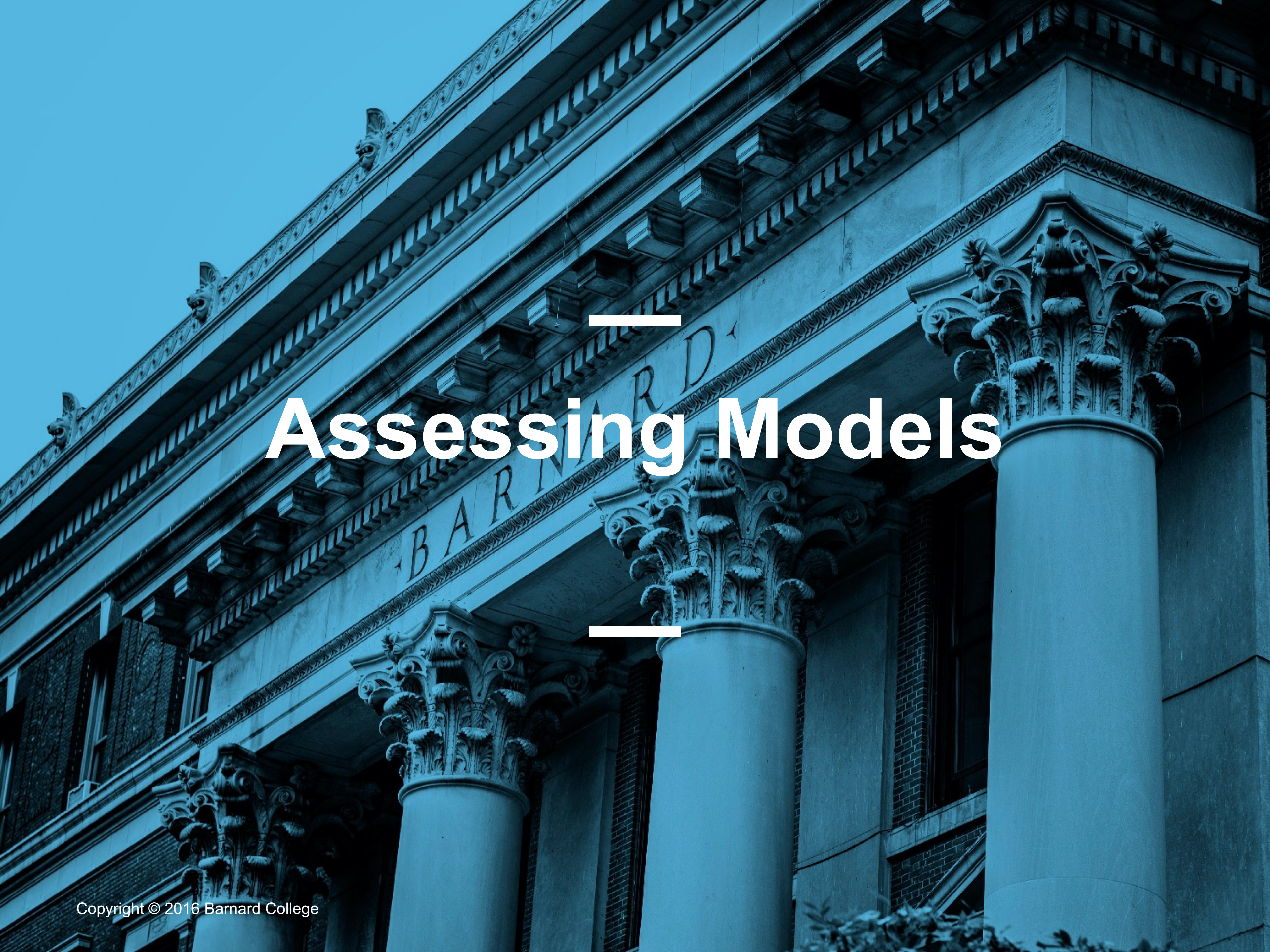


Hypothesis Testing

Choosing Between Two Viewpoints

- Based on data:
  - “Chocolate has no effect on cardiac disease”
  - “Yes, it does”

- Questions that we will consider:
  - Were data was drawn?
  - How the data was drawn?
  - What can we conclude from the data?



Assessing Models

Models

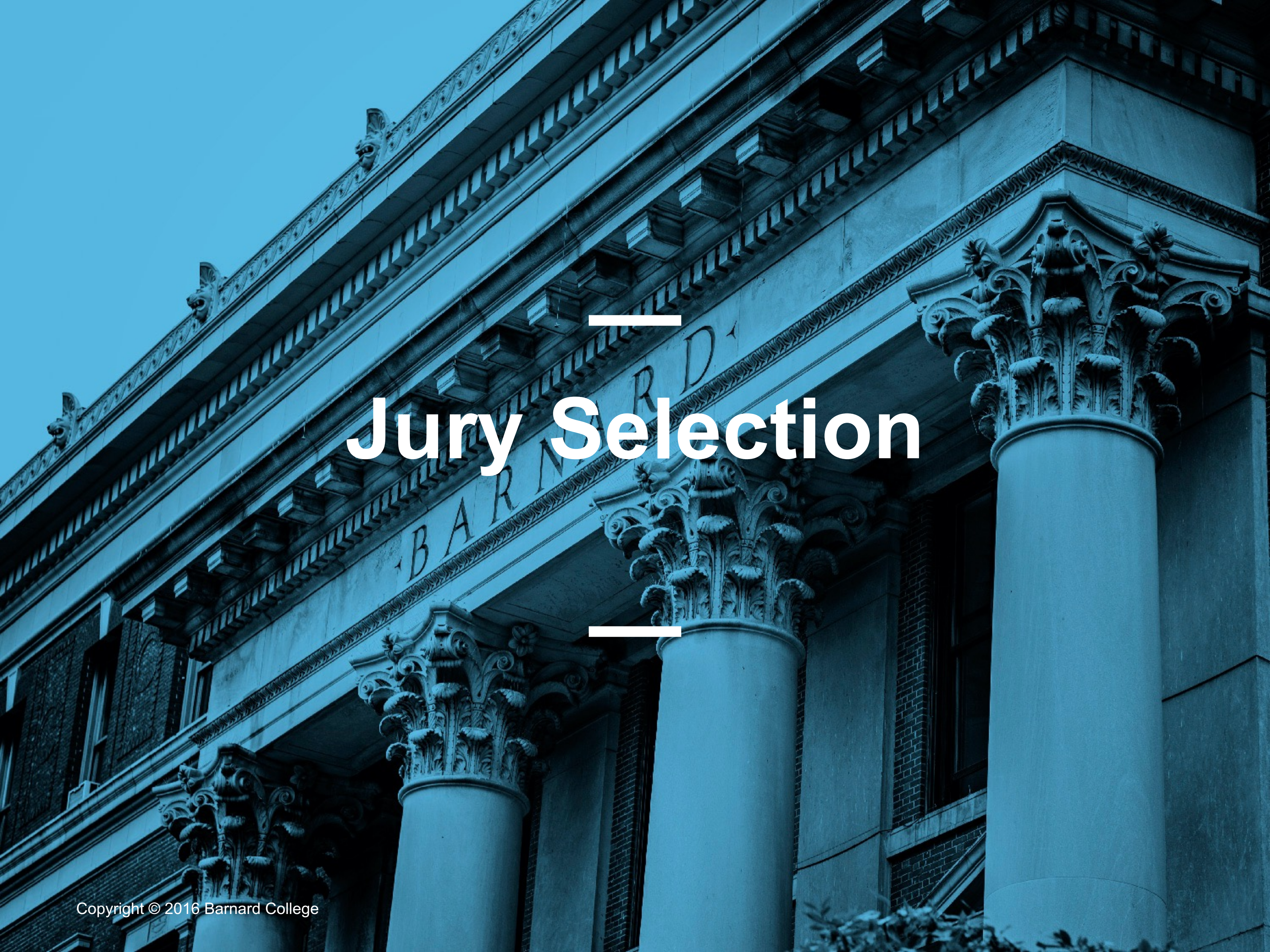
- A model is a set of assumptions about the data
- In data science, many models involve assumptions about processes that involve randomness:
  - “Chance models”
- **Key question:** does the model fit the data?

Approach to Assessing Models

- If we can simulate data according to the assumptions of the model, we can learn what the model predicts

- We can compare the model's predictions to the observed data

- If the data and the model's predictions are not consistent, that is evidence against the model



Jury Selection

Swain vs. Alabama, 1965

- Talladega County, Alabama
- Robert Swain, black man convicted of crime
- Appeal: one factor was all white-jury
- Only men 21 years or older were allowed to serve
- 26% of this population were black
- Swain’s jury panel consisted of 100 men
- 8 men on the panel were black

Supreme Court Ruling [in English]

- About disparities between the percentages in the eligible population and the jury panel, the Supreme Court wrote:
  - "… the overall percentage disparity has been small and reflects no studied attempt to include or exclude a specified number of Negros"
- Supreme Court denied Robert Swain's appeal

Supreme Court Ruling [in Data]

- **Paraphrase**: 8/100 is less than 26%, but not different enough to show Black men were systematically excluded

- **Question:** is 8/100 a realistic outcome if the jury panel selection process were truly unbiased?

Sampling from a Distribution

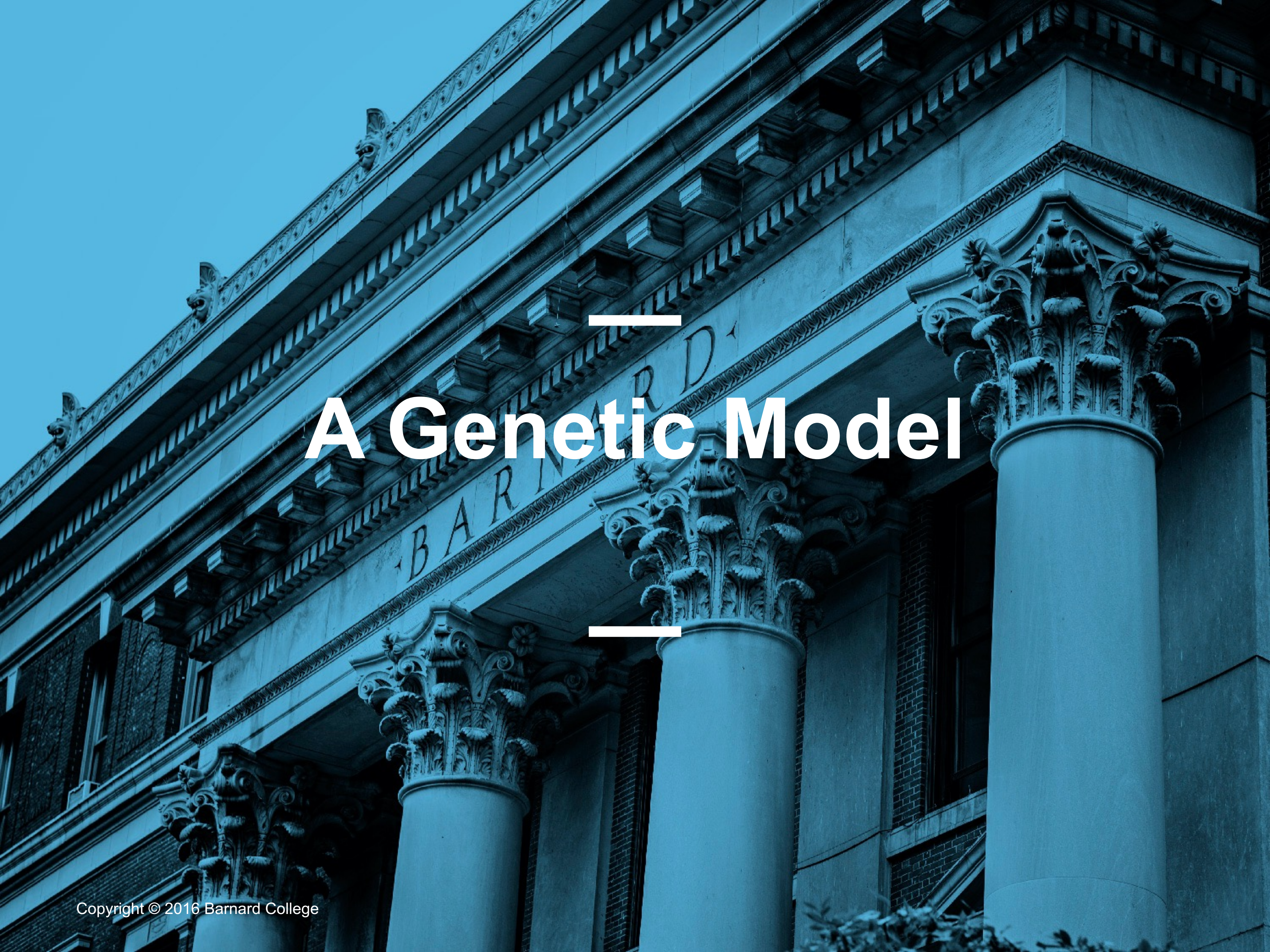
- Sample at random from a categorical distribution

```
sample_proportions(sample_size, pop_distribution)
```

- Samples at random from the population
  - Returns an array containing the distribution of the categories in the sample

Steps in Assessing a Model

- Choose a statistic that will help you decide whether the data support the model or an alternative view of the world
- Simulate statistic under the assumptions of the model
- Draw a histogram of the simulated values
  - This is the model's prediction for how the statistic should come out
- Compute the statistic from the sample in the study
  - If the two are not consistent => evidence against the model
  - If the two are consistent => data supports the model ***so far***



A Genetic Model

Mendel’s genetic model

- Pea plants of a particular kind
- Each one has either purple flowers or white flowers

- Mendel’s model:
  - Each plant is purple-flowering with chance 75%, regardless of the colors of the other plants
- Question:
  - Is the model good or not?

Choose a Statistic

- Take a sample, see what percent are purple-flowering
- If that percent is much larger or much smaller than 75, that is evidence against the model
- ***Distance*** from 75 is key
- Statistic:
  - | sample percent of purple-flowering plants – 75 |
- If the statistic is large, that is evidence against the model

Model and Alternative

- Jury Selection:
  - **Model:** The people on the jury panels were selected at random from the eligible population
  - **Alternative viewpoint**: No, they weren’t

- Genetics:
  - **Model:** Each plant has a 75% chance of having purple flowers
  - **Alternative viewpoint:** No, it doesnt

Steps in Assessing a Model

- Choose a statistic to measure the “discrepancy” between model and data
- Simulate the statistic under the model’s assumptions
- Compare the data to the model’s predictions:
  - Draw a histogram of simulated values of the statistic
  - Compute the observed statistic from the real sample
- If the observed statistic is far from the histogram, that is evidence against the model

Homework

- Reading 11.2 on your own
  - Multiple Categories

- Tomorrow’s lecture:
  - 11.3 – 11.4
  - A/B Testing (Chapter 12)