BC COMS 1016:
Intro to Comp Thinking & Data Science

Lecture 20 –
Correlation
Linear Regression

Announcements

- Project 2:
  - due Monday 04/18
- Lab 8:
  - Due Monday 04/18
- Homework 8 - Regression
  - Due Monday 04/18
- Dropping 1 homeworks and 1 lab

Remaining Assignments

- 3 more HWs:
  - HW08, HW09, HW10
- 1 more project:
  - Project 3 – Classification
    - working with movie scripts
- 2 more labs:
  - Lab08 – this week
  - Lab09 – last week

Data Science in this course

- Exploration
  - Discover patterns in data
  - Articulate insights (visualizations)
- Inference
  - Make reliable conclusions about the world
  - Statistics is useful
- Prediction
  - Informed guesses about unseen data

Correlation

Prediction

- To predict the value of a variable:
  - Identify (measurable) attributes that are associated with that variable
  - Describe the relation between the attributes and the variable you want to predict
  - Then, use the relation to predict the value of a variable

Visualizing Two Numerical Variables

- Trend
  - Positive association
  - Negative association
- Pattern
  - Any discernible “shape” in the scatter
  - Linear
  - Non-linear

Visualize, then quantify

The Correlation Coefficient *r*

- Measures **linear** association
- Based on standard units
- $-1 \leq r \leq 1$
  - $r = 1$: scatter is perfect straight line sloping up
  - $r = -1$: scatter is perfect straight line sloping down
- $r = 0$: No linear association; *uncorrelated*

Definition of *r*

Correlation Coefficient (r) =

average of product of standard(x) and standard(y)

Steps: 4 3 2 1

Measures how clustered the scattered data are around a straight line

Operations that leave *r* unchanged

R is not affected by:

- Changing the units of the measurement of the data
  - Because *r* is based on standard units
- Which variable is plotted on the x- and y-axes
  - Because the product of standard units is the same

Interpreting *r*

Causal Conclusion

Be careful …

- Correlation measures linear association
- Association doesn't imply causation
  - Two variables might be correlated, but that doesn't mean one causes the other

Nonlinearity and Outliers

Both can affect correlation

- Draw a scatter plot before computing *r*

Ecological Correlation

- Correlations based on groups or aggregated data

- Can be misleading:
  - For example, they can be artificially high

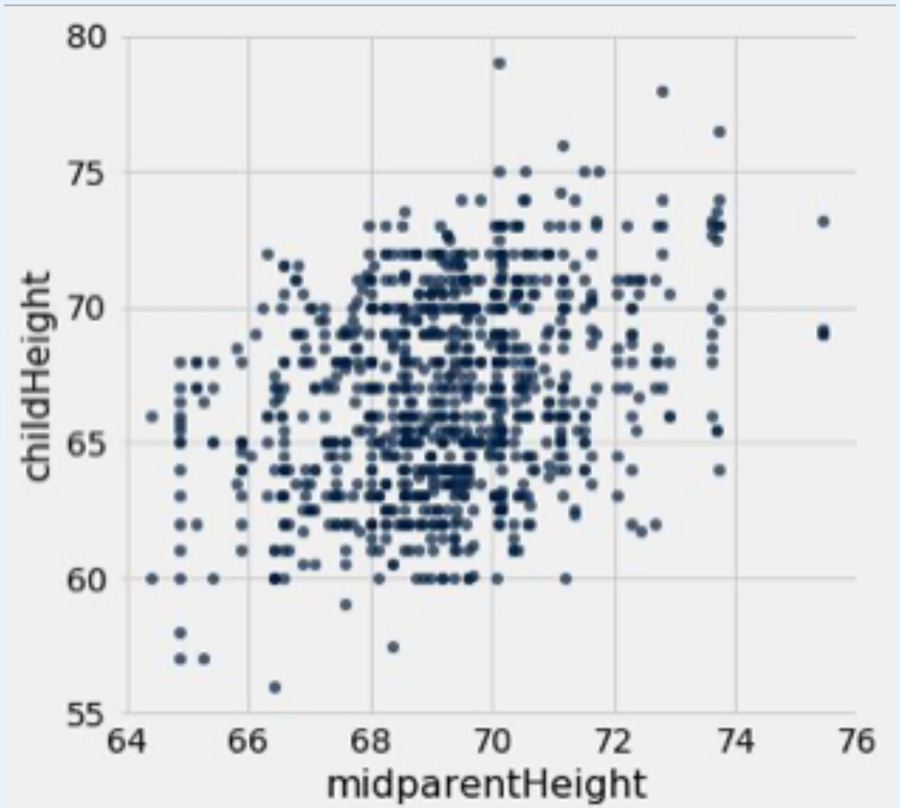
Prediction

Guess the future

- Based on incomplete information

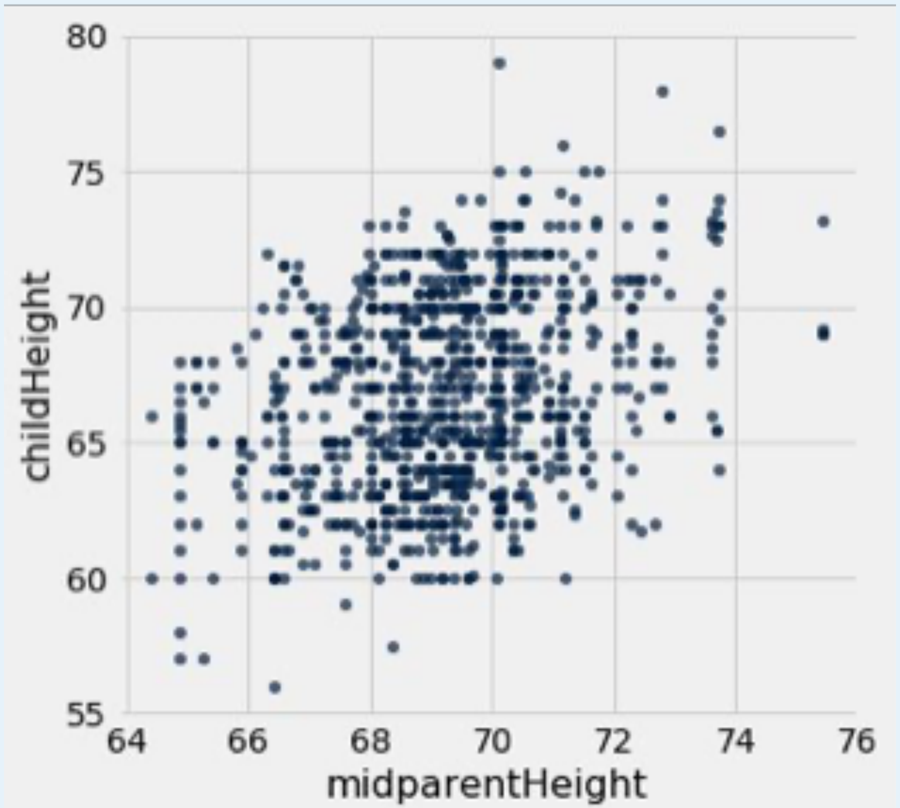
- One way of making predictions:
  - To predict an outcome for an individual,
  - find others who are like that individual
  - and whose outcomes you know.
  - Use those outcomes as the basis of your prediction.

Galton's Heights



Goal: Predict the height of a new child, based on that child's midparent height

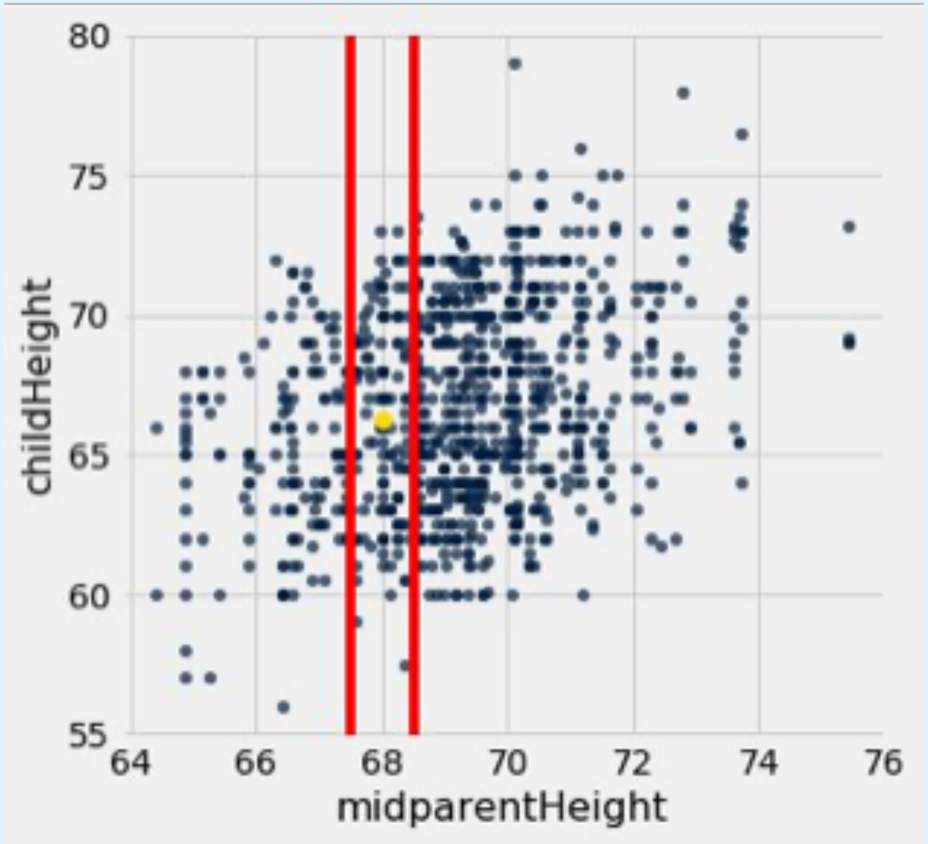
Galton's Heights



How can we predict a child's height given a midparent height of 68 inches?

Idea: Use the average height of the children of all families where the midparent Height is close to to 68 inches

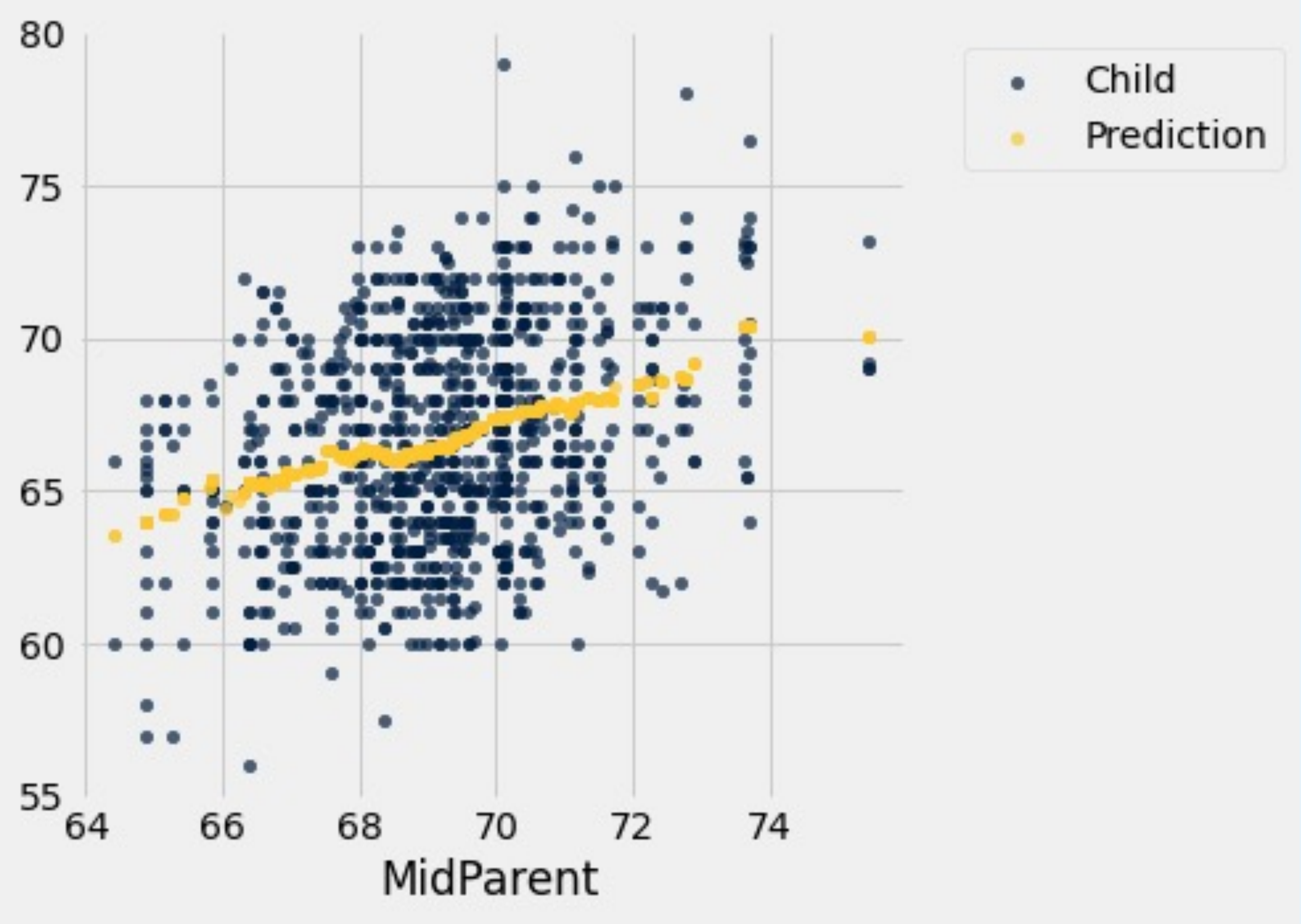
Galton's Heights



How can we predict a child's height given a midparent height of 68 inches?

Idea: Use the average height of the children of all families where the midparent Height is close to to 68 inches

Predicted Heights



Graph of Average

For each x value, the prediction is the average of the y values in its nearby group.

The graph of these predictions is the

graph of averages

If the association between x and y is linear, then points in the graph of averages tend to fall on a line. The line is called the **regression line**

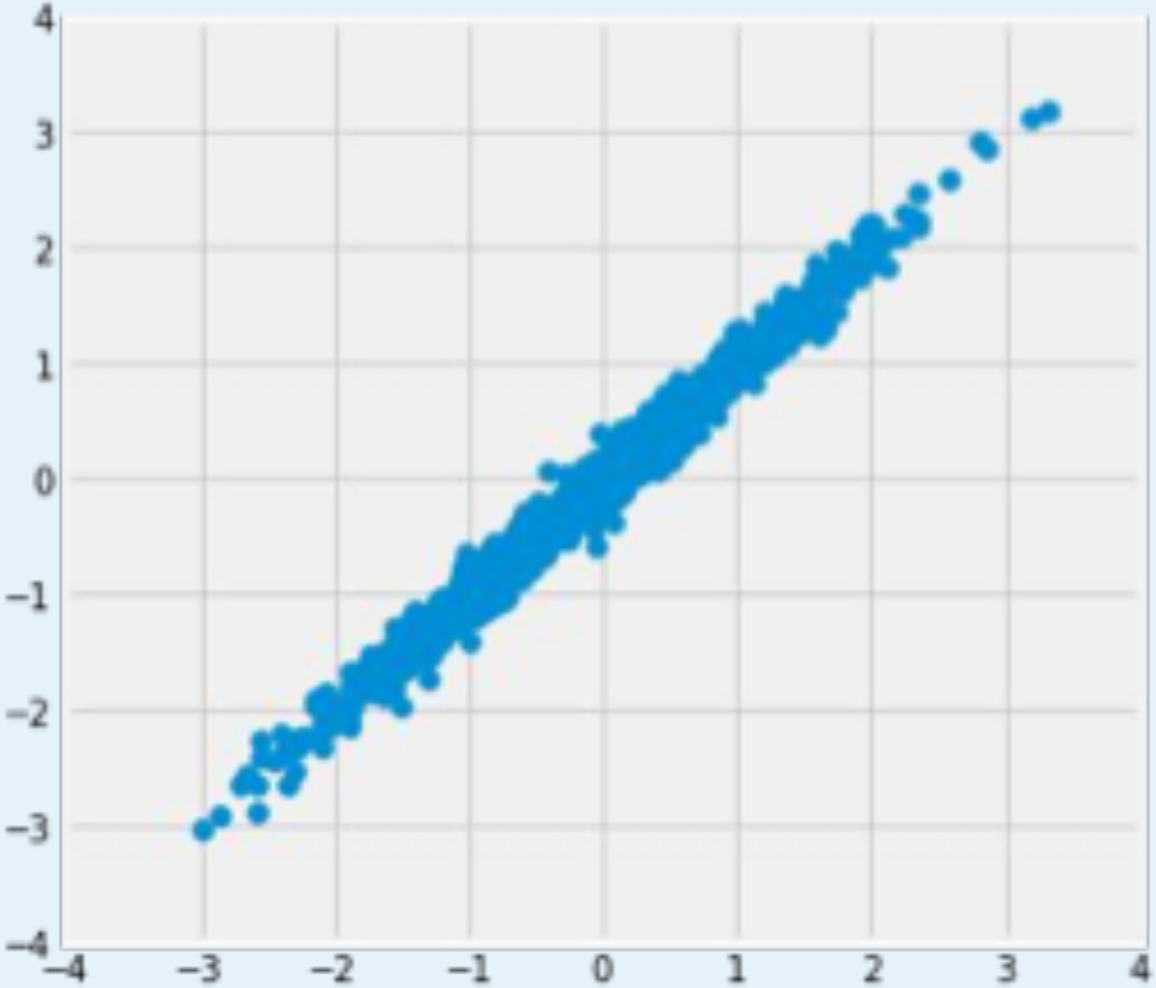
Nearest Neighbor Regression

A method for predicting a numerical y,
given a value of x:

- Identify the group of points where the values of x are close to the given value
- The prediction is the average of the y values for the group

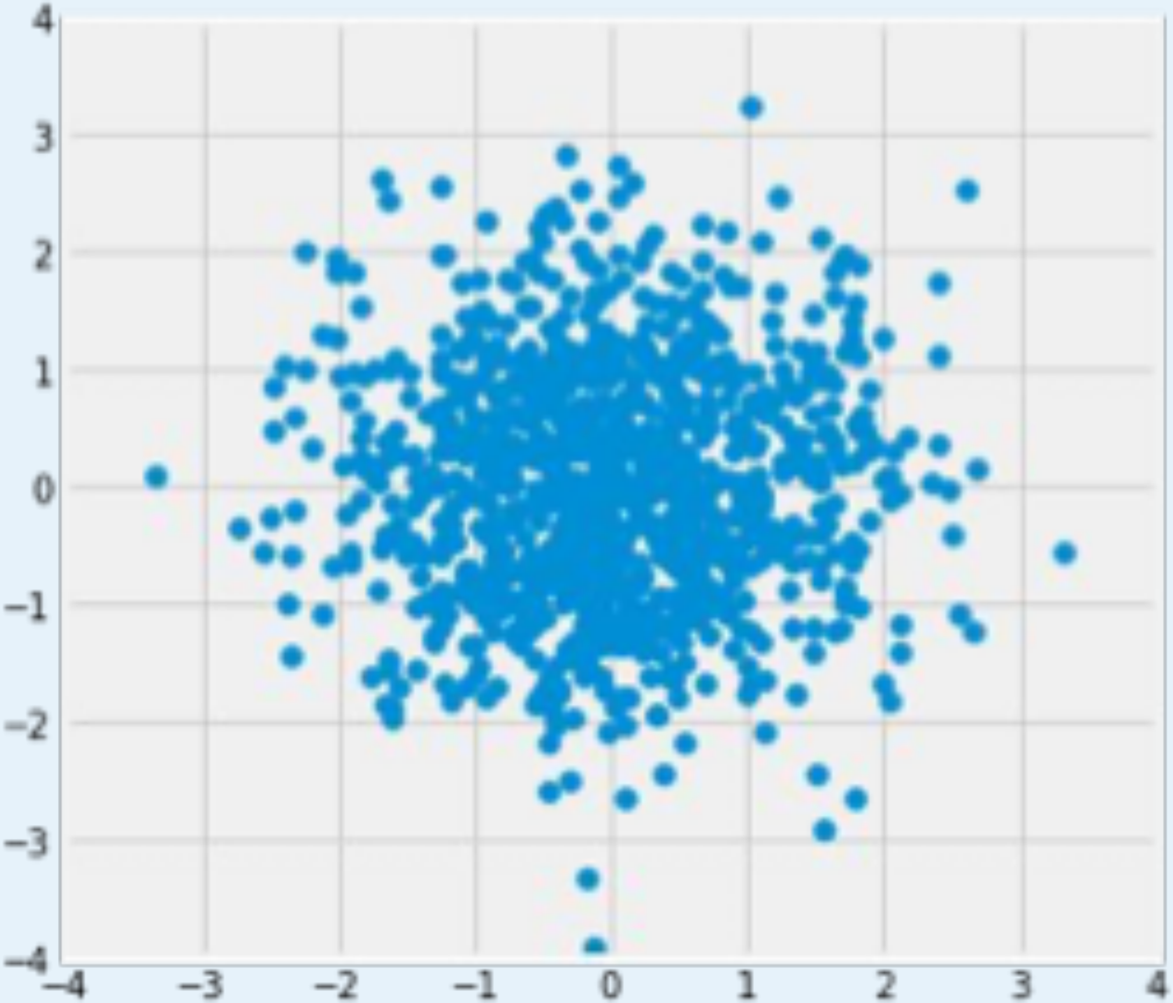
Linear Regression

Where is the prediction line?



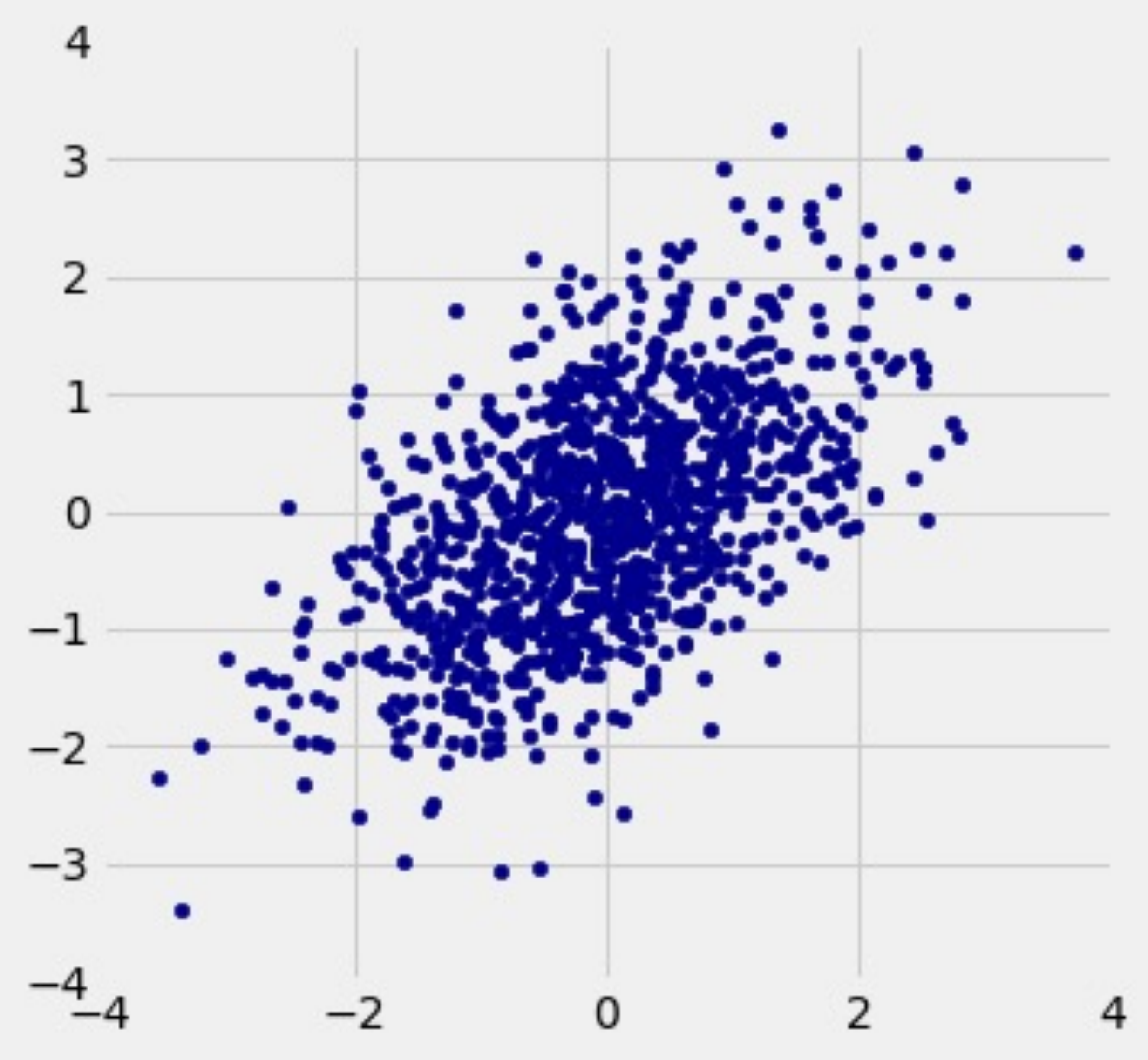
$r = 0.99$

Where is the prediction line?



$r = 0.0$

Where is the prediction line?



$r = 0.5$

Identifying the Line

- If the scatter plot is oval shaped, then we can spot an important feature of the regression line

Linear Regression

A statement about x and y pairs

- Measured in *standard units*
- Describing the deviation of x from 0 (the average of x's)
- And the deviation of y from 0 (the average of y’s)

On average,

y deviates from 0 less than x deviates from 0

$$y_{su} = r \times x_{su}$$

Slope and Intercept

Regression Line Equation

In original units, the regression line has this equation:

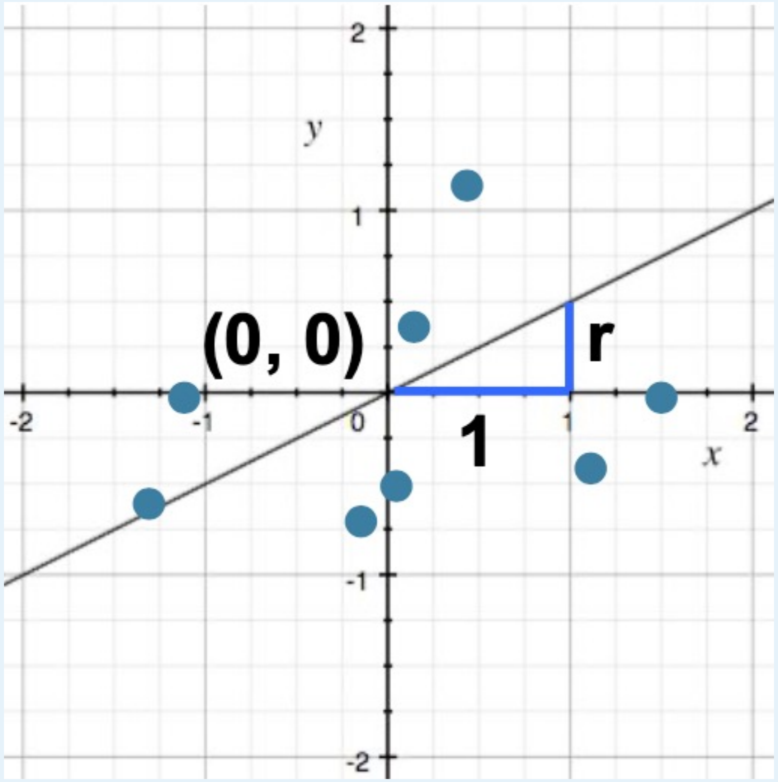
$$\frac{estimate\ of\ y\ - mean(y)}{SD\ of\ y} = r \times \frac{given\ x\ - mean(x)}{SD\ of\ x}$$

Lines can be expressed by *slope* & *intercept*

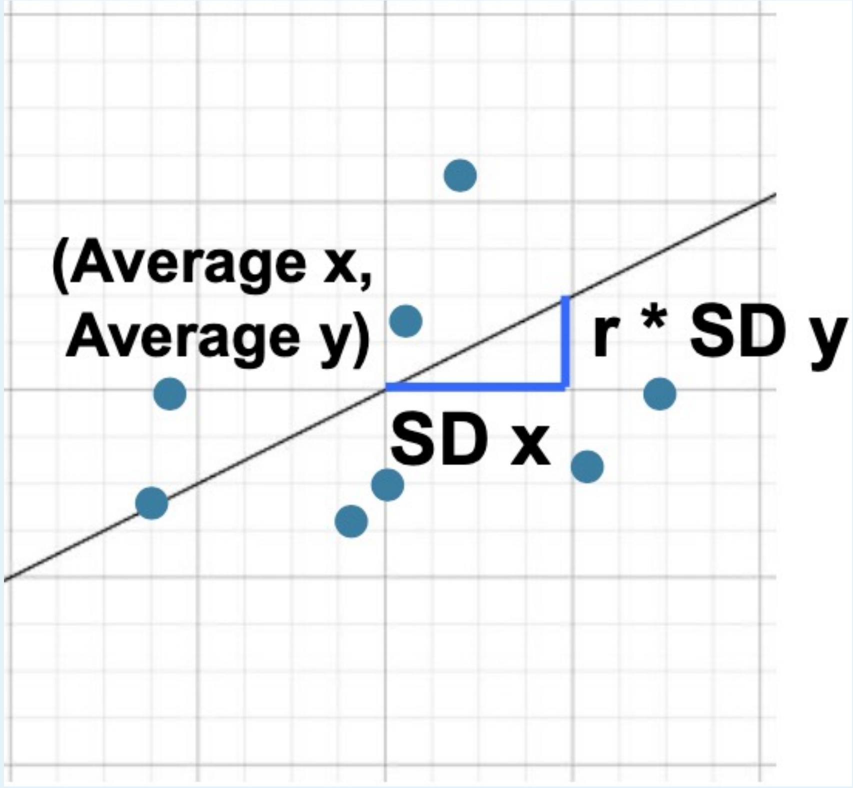
$$y = slope \times x + intercept$$

Regression Line

Standard Units



Original Unites



Slope and Intercept

$$estimate\ of\ y = slope\ * x + intercept$$

$$\boldsymbol{slope\ of\ the\ regression\ line}$$

$$r\ *\ \frac{SD\ of\ y}{SD\ of\ x}$$

$$\boldsymbol{intercept\ of\ the\ regression\ line}$$

$$mean(y) - slope \times mean(x)$$

Prediction with Linear Regression

Goal: Predict *y* using *x*

Examples:

- Predict # *hospital beds available* using *air pollution*
- Predict *house prices* using *house size*
- Predict # *app users* using # app downloads

Regression Estimate

Goal: Predict *y* using *x*

To find the regression estimate oy *y*:

- Convert the given *x* to standard units
- Multiply by *r*
- That's the regression estimate of *y*, but:
  - It's in standard units
  - So convert it back to the original units of *y*

Regression Line Equation

In original units, the regression line has this equation:

$$y_{su} = r \times x_{su}$$

$$\frac{estimate\ of\ y\ - mean(y)}{SD\ of\ y} = r \times \frac{given\ x\ - mean(x)}{SD\ of\ x}$$

Lines can be expressed by *slope* & *intercept*

$$y = slope \times x + intercept$$

What we want

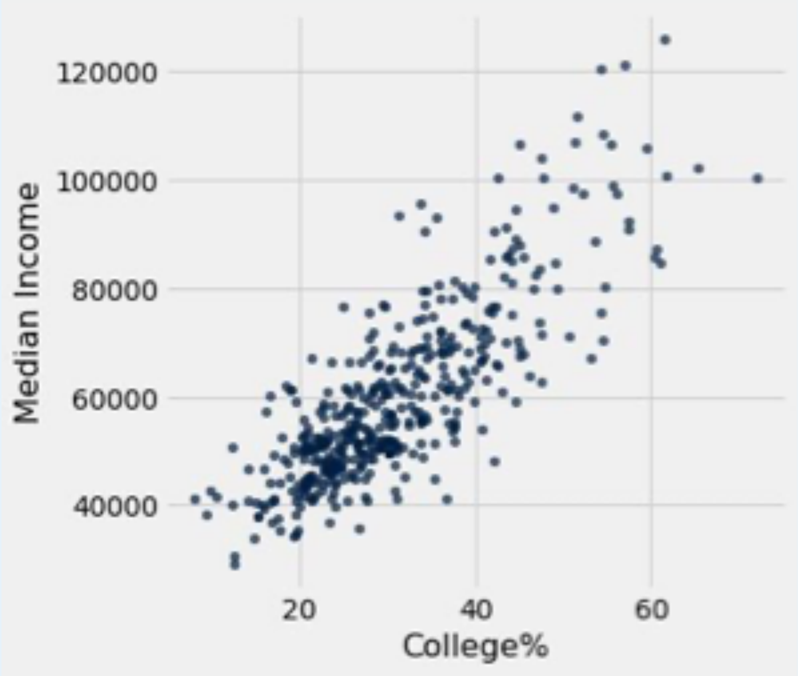
What we observe

Discussion Question

Based only on the graph, which must be true?

1. Going to college causes people to earn more.
2. For any district, having more college-educated people live there causes median incomes to rise.
3. For any district, having a higher median income causes more college-educated people to move there.

USA Congressional Districts 2016



Least Squares

Error in Estimation

- **error = actual value – estimate**
- Typically, some errors are positive and some are negative
- To measure the rough size of the errors
  - **square** the **errors** to eliminate cancellation
  - Take the **mean** of the squared errors
  - Take the square **root** to fix the units
- **Root mean square error** (rmse)

Least Squares Line

- Minimized the root mean squared error among all lines

- Equivalently, minimizes the mean squared error among all lines

- Names:
  - “Best fit” line
  - Least squares line
  - Regression line

Numerical Optimization

- Numerical minimization is approximate but effective
- Lots of machine learning uses numerical minimization (demo)
- If the function **mse(a, b)** returns the mse of estimation using the line "estimate = a*x* + b",
  - then **minimize(mse)**returns array [a0, b0]
  - a0 is the slope and b0 the intercept of the line that *minimizes* the mse among lines with arbitrary slope a and arbitrary intercept b (that is, among all lines)

Residiuals

- Error in regression estimate
- One residual corresponding to each point (x, y)
- **residual**
  = observed *y* - regression estimate of *y*
  = observed y - height of regression line at x
  = vertical distance between the point and the best line

Residual Plot

A scatter diagram of residuals

- Should look like an unassociated blob for linear relations
- But will show patterns for non-linear relations
- Used to check whether linear regression is appropriate
- Look for curves, trends, changes in spread, outliers, or any other patterns

Properties of residuals

- Residuals from a linear regression **always** have
  - Zero mean
    - (so rmse = SD of residuals)
  - **Zero** correlation with x
  - **Zero** correlation with the fitted values
- These are all true **no matter what the data look like**
  - Just like deviations from mean are zero on average